Let k and $\sigma = \exp(-\gamma d)$ denote the voltage reflection coefficients corresponding to Z_K and Z_N, respectively. The voltage reflection coefficients for all points are simple functions of k and σ.

In some cases, the matrix element is found by determining the impedance Z corresponding to the element as a voltage reflection coefficient Γ. The impedance is plotted on a transmission-line chart, and the value of Γ is read off the chart to obtain the desired matrix element. If the impedance has a negative real component, or an angle outside the $-90°$ to $+90°$ range, an extended transmission-line chart[11] may be used. Since extended charts are not readily available, a conventional chart may be used by recalling that $Z(1/\Gamma) = -Z(\Gamma)$. One plots $-Z$, reads $c = 1/\Gamma$, and computes $\Gamma = 1/c$.

The elements of the Z matrix are given by

$$Z_{11} = Z_{22} = Z_0 \coth \gamma d = Z_0 Z(\sigma^2) = Z_0 Z_M$$

$$Z_{12} = Z_{21} = Z_0/\sinh \gamma d$$

The term $\sinh \gamma d$, which appears in other matrix elements, is given by

$$Z(j \sinh \gamma d) = -[Z(-j\sigma)]^2 = -Z_R^2$$

The elements of the matrix are given by

$$Y_{11} = Y_{22} = \coth \gamma d/Z_0 = Z_M/Z_0$$

$$Y_{12} = Y_{21} = 1/Z_0 \sinh \gamma d$$

The elements of the $ABCD$ matrix are given by

$$Z(A) = Z(D) = \cosh \gamma d = -[Z(\sigma)]^2$$

$$= -Z_N^2$$

$$B = Z_0 \sinh \gamma d$$

$$C = \sinh \gamma d/Z_0$$

The scattering matrix elements are given by

$$Z(S_{11}) = Z(S_{22}) = Z(k)/Z(k\sigma^2) = Z_K Z_U$$

$$= Z_U/Z_J$$

and

$$Z(S_{12}) = Z(S_{21}) = Z(\sigma)/Z(k^2\sigma) = Z_N/Z_X$$

The less known r matrix is defined by

$$\begin{pmatrix} E_{12} \\ E_{11} \end{pmatrix} = \begin{pmatrix} r_{11} & r_{12} \\ r_{21} & r_{22} \end{pmatrix} \begin{pmatrix} E_{22} \\ E_{21} \end{pmatrix}$$

The elements of this matrix are given by

$$Z(r_{11}) = Z(\sigma)/Z(k^2/\sigma) = Z_N Z_W$$

$$Z(r_{12}) = Z(-k\sigma)Z(k/\sigma) = Z_Y Z_S$$

$$r_{21} = -r_{12}$$

$$Z(r_{22}) = -Z(\sigma)/Z(k^2\sigma) = -Z_N/Z_X$$

H. F. Mathis
Dept. of Elec. Engrg.
The Ohio State University
Columbus, Ohio

[11] Mathis, H. F., Extended transmission-line charts, *Electronics*, vol 33, Sep 23, 1960, pp 76, 78.

Note on Tetrahedral Junction

Weiss [1] has described a reactive waveguide switch consisting of a ferrite rod at the junction of two mutually cross-polarized rectangular waveguides, as shown in Fig. 1. When the ferrite rod is unmagnetized, the system represents a transition to cutoff waveguide. When the ferrite rod is longitudinally magnetized, the RF energy is coupled to the crossed waveguide.

Weiss [2] has noted that the modes of propagation on the tetrahedral junction are the same as those on the Reggia and Spencer phase shifter [3]. He has described the useful operating range of these devices as extending from the onset of the dielectric waveguide effect to the onset of elliptical Faraday rotation. In this range, the Weiss model indicates that the composite ferrite air waveguide can support only a single elliptically polarized mode of propagation. As the ferrite rod is increased above a critical diameter, a second ellipitically polarized mode becomes propagating and the system exhibits elliptical Faraday rotation, that is, rotation is suppressed in the range of ferrite diameters for which only a single elliptically polarized mode is propagating.

It is the purpose of this note to describe the tetrahedral junction in terms of the single-tapered mode coupler. However, we shall first briefly compare mode interference couplers with those that are single tapered.

Miller [4] has shown that when two transmission lines with equal-phase velocities are continuously coupled over some length, power introduced into one line will be completely transferred periodically back and forth between them. Such a system can generally support two forward modes of propagation, which have equal amounts of power in each line and which propagate with different velocities through the structure. One is called the even mode, because the electric field is in the same direction in each line; the other is called the odd mode, because the fields are oppositely oriented. Power transfer is effected by the interference between these modes, and this type of coupler is therefore known as a mode interference coupler; in general, it is frequency dependent. This is what occurs in Faraday structures where the normal modes are circularly polarized. If power is introduced into both lines to correspond to one of the normal modes, no coupling will take place and the power will be emergent in the same normal mode. If the two coupled modes have a fixed-phase difference between them, the power transfer is still periodic, but now the normal modes have unequal powers in the two lines and the power division is dependent on the ratio of velocity difference to coupling factor.

More recently Cook [5] has shown that if the phase velocity difference between the two coupled lines is made grossly different at one end of the coupled lines, equal in the center, and again unequal in the opposite sense, complete power transfer with only a residual power fluctuation is effected, which, moreover, is independent of coupling factor and frequency, provided the coupling length is long. When this is so, excitation of one of the coupled lines corresponds to all the power in one of the forward normal modes of propagation, and the power division is simply dependent on the final-phase difference between the coupled lines. In such a system, one of the normal modes of propagation is discriminated against, and power transfer between the coupled lines occurs with only a single forward normal mode of propagation.

Manuscript received October 2, 1964.

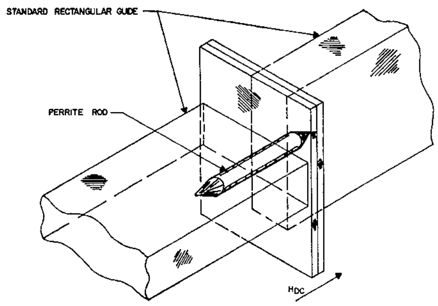

Fig. 1. The tetrahedral junction.

Fox [6] has enlarged on Cook's scheme and has shown that by varying both the velocities of the coupled lines and the coupling factor, the residual power fluctuation is eliminated. In particular, the difference in phase velocity is made to vary cosinusoidally and the coupling factor sinusoidally.

In mode interference couplers, the phase relation between the electric fields in the two lines is 90°. However, for this medium, the phase difference is 0° or 180°, depending on whether the power is introduced in the line having the higher or lower phase velocity; hence the coupled modes are the normal modes for the local cross section.

In the switch described by Weiss, the system represents a transition to cutoff waveguide when the ferrite is unmagnetized. At the plane of symmetry, the waveguide boundary conditions impose a sharp crossover of the phase velocities of the coupled modes. When the ferrite rod is magnetized, this phase distribution corresponds to a zero-dB coupler and the energy is accepted by the rotated waveguide. We also note that the wave is circularly polarized at the plane of symmetry.

Because the phase transition between the coupled modes is discontinuous, both normal modes are excited. However, one of these is cut off; hence no beat interference results. If at the same time the backward propagating mode is matched out, the power is transmitted past the junction.

More generally, the tetrahedral junction is only one example of several zero-dB couplers that rely on the single-tapered mode coupling theory in which one of the normal modes is cut off.

A second example was realized by taking an X-band 3-dB sidewall hybrid and placing metal inserts into the coupling region, as shown in Fig. 2. In this manner the TE_{20} mode, which is one of the normal modes of the hybrid, was cut off. The insertion loss for this structure between ports 1 and 2 was

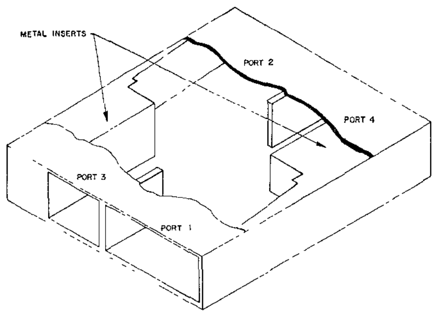

Fig. 2. Zero-dB coupler.

0.15 dB. No loads were required on ports 3 and 4, and no external matching was necessary.

A third application of the principle would consist of a compact round-to-rectangular waveguide transition.

J. HELSZAJN
Microwave Associates, Inc.
Burlington, Mass.

REFERENCES

[1] Weiss, J. A., Tetrahedral junction, *J. Appl. Phys.*, Suppl, vol 31, 1960, pp 1685–1695.
[2] ——, A phenomenological theory of the Reggia-Spencer phase shifter, *Proc. IRE*, vol 47, Jun 1959, pp 1130–1137.
[3] Reggia, F., and E. G. Spencer, A new technique in ferrite phase shifting for beam scanning of microwave antennas, *Proc. IRE*, vol 45, Nov 1957, pp 1510–1517.
[4] Miller, S. E., Coupled wave theory and waveguide application, *Bell Sys. Tech. J.*, vol 32, 1954, pp 661–719.
[5] Cook, J. S., Tapered velocity couplers, *Bell Sys. Tech. J.*, vol 34, 1955, pp 807–822.
[6] Fox, A. G., Wave coupling by warped normal modes, *Bell Sys. Tech. J.*, vol 34, 1955, pp 823–852.

Pulse Power Capacity of Short-Slot Couplers

Manuscript received October 7, 1964. This work was performed under Contract No. AF 30(602)-3174 for the Rome Air Development Center.

A short-slot sidewall coupler and a short-slot topwall coupler were both tested at S band for their pulse power capacity. Both types of coupler were tested in the 0-dB coupler configuration [1], which had the practical convenience of requiring only one high-power load.

In order to make meaningful tests, a special test section of S-band waveguide was constructed. This test section had the standard WR-284 waveguide width of 2.840 inches, but was only 0.447-inch high, which is one third of the standard waveguide height of 1.340 inches. A two-section waveguide transformer was placed at each end of the test section to match it to standard waveguide; the test section, including both transformers, was measured to have a maximum VSWR of 1.03 over the frequency band 2.7 to 2.9 Gc/s. All the measurements were made at a frequency of 2.856 Gc/s. The VSWR of the high-power water load at this frequency was better than 1.05. The pulse length used throughout the tests was 3 μs.

The test procedure in each case was to increase the klystron power until arcing occurred in the waveguide. The klystron power was then reduced until arcing stopped, and this power level was maintained for 10 minutes without any further arcing. Taking the test section, the sidewall coupler, and the topwall coupler in turn, the all-clear pluse power was 1.25, 2.72, and 1.67 MW, respectively. (The corresponding all-clear power averages were 0.431, 0.94, and 0.576 kW, respectively.)

Given the fact that the test section is only one third of the standard waveguide height, the pulse power capacity of the short-slot sidewall coupler is approximately 72 per cent of WR-284, and the pulse power capacity of the short-slot topwall coupler is approximately 44 per cent of WR-284 waveguide.

ACKNOWLEDGMENTS

The authors wish to acknowledge the assistance of R. B. Larrick and P. R. Reznek in making the measurements.

The high-power tests were conducted at the Eitel-McCullough plant in San Carlos, Calif., with the cooperation of Dr. G. Caryotakis.

L. YOUNG
B. M. SCHIFFMAN
Electromagnetic Techniques Lab.
Stanford Research Institute
Menlo Park, Calif.

REFERENCE

[1] Young, L, Waveguide 0-dB and 3-dB directional couplers as harmonic pads, *Microwave J.*, vol 7, Mar 1964, pp 79–87.

Accurate Measurements for Dynamic Representations of Parametric Amplifier Varactor Diodes

Manuscript received September 21, 1964; revised October 6, 1964.

Varactor diode parameter representations useful for describing circuit performance are usually derived from UHF or microwave measurements. Although many measurement methods have been used to date, the two main classes appear to be measurements which result in "static" and "dynamic" diode representations.

"Static" representations are defined here as those in which a UHF or microwave signal of small amplitude is used to measure characteristics of the diode which can then be expressed in terms of an equivalent circuit. UHF bridge and Q-meter [1], [2], microwave junction [3], and microwave cavity [4] measurements have been used to obtain equivalent circuits. Smith-chart data obtained from measured microwave VSWR data obtained as a function of diode bias [5], [6] or frequency [7] has been used to arrive at equivalent circuit parameters for the diode junction or cartridge. Measurements yielding circuit parameters as polynomial functions of frequency have been developed [8].

"Dynamic" representations are defined as those which characterize the diode in terms of a parameter which expresses the diode's performance under pumping conditions. It is the purpose of this note to propose a rapid and accurate measurement scheme for obtaining a dynamic quality parameter applicable to the commonly used parametric amplifier varactor.

A dynamic parameter ω_d is defined as the ratio of the total elastance variation to four times the series resistance of a pumped parametric amplifier diode [8]: $\omega_d = \Delta(1/C)/4R$. The extremes of elastance variation correspond to definite values of diode capacitance, such that if C_1 and C_2 are respectively the minimum and maximum diode capacitance, ω_d becomes

$$\omega_d = 2\pi f_d = \frac{1}{4R}\left(\frac{1}{C_1} - \frac{1}{C_2}\right) \tag{1}$$

where f_d is the "quality parameter." Let diode impedances Z_1 and Z_2 correspond to diode conditions which produced capacitances C_1 and C_2. If Z_1 and Z_2 are valid at frequency f, then

$$1 + j\,(f/2f_d) = (Z_1 + Z_2^*)/(Z_1 - Z_2) \tag{2}$$

where the asterisk denotes the complex conjugate. The measurement of Z_1 and Z_2 at the diode is not so easily accomplished, but measurements may be made when the diode terminates a transmission line. Assume that a lossless transformer is inserted between the diode and the transmission line. At the input to the transformer, the reflection coefficients Γ_1 and Γ_2 corresponding to impedances Z_1 and Z_2 may then be defined at frequency f, and f_d may be written [9] as

$$f_d = \frac{f}{2}\left[\left|\frac{1 - \Gamma_1\Gamma_2^*}{\Gamma_1 - \Gamma_2}\right|^2 - 1\right]^{-1/2} \tag{3}$$

Equation (3) can be rewritten in terms of the magnitudes γ_1 and γ_2, and of the phase difference Ψ between Γ_1 and Γ_2:

$$f_d = \frac{f}{2}\left[\frac{\gamma_1^2 - \gamma_2^2 - 2\gamma_1\gamma_2\cos\Psi}{(1 - \gamma_1^2)(1 - \gamma_2^2)}\right]^{1/2} \tag{4}$$

Now if the lossless transformer is adjusted such that either γ_1 and γ_2 is zero, (4) can be simplified and made independent of the phase Ψ; setting $\gamma_2 = 0$ results in

$$f_d = \frac{f}{2}\left[\frac{1}{\gamma_1^2} - 1\right]^{-1/2} \tag{5}$$

The measurement of f_d is now reduced to the measurement of a single reflection coefficient magnitude at a single frequency. For practical values of f, f_d is usually large ($f_d \gg f$) such that γ_1 is large and difficult to measure accurately using standard slotted-line techniques. However, if a reflectometer is used